

P. رائد قنبر
 احمد / رائد / قنبر
 10:00 - 11:00

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Palestinian National Authority Ministry of High Education Palestine Technical University Name: <u>Maen Q'dan</u> <u>ممن قنبر</u>	بسم الله الرحمن الرحيم 	Numerical Analysis Wed 16/11/2011 Period: 1 Hour First Exam
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Question 1: ~~Circle the correct answer~~ (10 points)

Circle the correct answer:

1) The five-digit chopping arithmetic of $\sqrt{3}$ is:

- ☒ a) 0.17320×10^1
 b) 0.17321×10^1
 c) 0.17320×10^{-1}
 d) 0.17321×10^{-1}

2) The third Taylor polynomial for $f(x) = \cos x$ about $x_0=0$:

- a) $1 - x - \frac{x^2}{2} + \frac{x^3}{6}$
☒ b) $1 - \frac{x^2}{2}$
 c) $-x + \frac{x^3}{6}$
 d) $-1 + \frac{x^2}{2} - \frac{x^3}{6}$

$$\begin{aligned}
 &P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 \\
 &\cos(0) - \sin(0)x - \frac{\cos(0)}{2}x^2 + \frac{\sin(0)}{6}x^3 \\
 &1 - 0 - \frac{1}{2}x^2 + 0 = 1 - \frac{x^2}{2}
 \end{aligned}$$

3) Number of iterations needed to solve $f(x)=0$ on $[0,1]$.

- ☒ a) 7
 b) 6
 c) 2
 d) 5

$$|P_n - P| \leq \frac{b-a}{2^n} \leq 10^{-2}$$

$$\frac{1-0}{2^n} \leq 10^{-2} \Rightarrow \frac{1}{2^n} \leq 10^{-2}$$

4) Let $g(x) = 1 + x - \frac{x^2}{4}$, the fixed point of g is:

- a) 4
 b) 1
☒ c) 2
 d) -1

$$\begin{aligned}
 g(4) &= 1 + 4 - \frac{16}{4} = 5 - 4 = 1 \\
 g(1) &= 1 + 1 - \frac{1}{4} = 2 - \frac{1}{4} = 1.75 \\
 g(2) &= 1 + 2 - \frac{4}{4} = 3 - 1 = 2
 \end{aligned}$$

5) Let $\alpha_n = \frac{n+3}{n^3}$, $n \geq 1$. The sequence $\{\alpha_n\}_{n=1}^{\infty}$ converges to zero with rate of

- a) $O(\frac{1}{n})$
☒ b) $O(\frac{1}{n^2})$
 c) $O(\frac{1}{n^3})$

$$\frac{1}{2^n} \leq 10^{-2}$$

$$\log 1 - \log 2^n \leq \log 10^{-2}$$

$$\log 0 - n \log 2 \leq -2 \log 10$$

$$-n \log 2 \leq -2 \log 10$$

$$n \log 2 \geq 2 \log 10$$

$$n \geq \frac{2 \log 10}{\log 2} \approx 6.64$$

(10 points)

Question 2:

Use the Bisection method to find the third approximation of $\sqrt[3]{7}$.

$$\sqrt[3]{7} = x \Rightarrow x^3 = 7 \Rightarrow x^3 - 7 = 0 \Rightarrow P(x) = x^3 - 7 = 0$$

$$\begin{aligned}
 P(1) &= -6 < 0 \\
 P(2) &= 1 > 0
 \end{aligned}$$

the period is $[1, 2]$

$$P_0 = \frac{1+2}{2} = \frac{3}{2} = 1.5 \Rightarrow P(1.5) = -3.625 < 0$$

\therefore the period become $[1.5, 2]$

$$P_1 = \frac{1.5+2}{2} = 1.75 \Rightarrow P(1.75) = -1.640625 < 0$$

\therefore the period become $[1.75, 2]$

$$P_2 = \frac{1.75+2}{2} = 1.875 \Rightarrow P(1.875) = -0.408203125 < 0$$

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the period become $[1.875, 2]$

$$p_3 = \frac{1.875 + 2}{2} = \boxed{1.9375} \Rightarrow f(1.9375) = 0.273193359 > 0$$

\therefore The ~~period~~ period become $[1.875, 1.9375]$

$\sqrt[3]{7}$ exist ~~between~~ in the period: $[1.875, 1.9375]$

Note:

The actual value of $\sqrt[3]{7} = 1.912931183$

Question 3:

(15 points)

Let $g(x) = 1 + x - \frac{x^2}{4}$

- Show that $g(x)$ has a unique fixed point on $[1, 3]$.
- Use starting value $P_0 = 1.6$ to find P_2 .
- Find the number of iterations needed to evaluate an approximation of P with accuracy of 10^{-6} .

① $g(x)$ is cont and diff on $(1, 3)$

is $g(a) \stackrel{??}{=} a \Rightarrow g(a) = g(1) = 1 + 1 - \frac{1}{4} = 2 - \frac{1}{4} = 1.75 \neq 1$

is $g(b) \stackrel{??}{=} b \Rightarrow g(b) = g(3) = 1 + 3 - \frac{3^2}{4} = 4 - \frac{9}{4} = 1.7778 \neq 3$

\therefore there exist a number in the period $[1, 3]$ ~~s.t. $g(c) = c$~~

s.t. $g(c) = c$

let take 2

$g(2) = 1 + 2 - \frac{(2)^2}{4} = 1 + 2 - \frac{4}{4} = 1 + 2 - 1 = 2$

$\therefore g(c) = c \Rightarrow g(2) = 2$

\therefore there really exist a c in the period $[1, 3]$ s.t. $g(c) = c$

② Starting using $P_0 = 1.6$

$P_1 = g(P_0) = g(1.6) = 1 + 1.6 - \frac{(1.6)^2}{4} = 2.6 - \frac{(1.6)^2}{4} = 2.6 - 0.64 = 1.96$

$P_2 = g(P_1) = g(1.96) = 1 + 1.96 - \frac{(1.96)^2}{4} = 2.96 - 0.9604 = 1.9996$

\therefore it is converge to 2 (to the fixed point).

③ $|P_n - P| \leq \frac{1}{2^n} (b-a) \leq 10^{-6} \Rightarrow \frac{(3-1)}{2^n} \leq 10^{-6}$

$\frac{2}{2^n} \leq 10^{-6} \Rightarrow \log\left(\frac{2}{2^n}\right) \leq \log 10^{-6} \Rightarrow \log 2 \times \log 2^n \leq \log 10^{-6}$

$\Rightarrow \log 2 - n \log 2 \leq \log 10^{-6} \Rightarrow -n \log 2 \leq \log 10^{-6} - \log 2$

~~$n \log 2 \geq 6 - 0.301029995$~~ $-n \log 2 \leq -6 - 0.301029995$

③

$$-n \log 2 \leq -5.698970004$$

$\times (-)$

$$n \log 2 \geq 5.698970004$$

$$n \geq \frac{5.698970004}{\log 2}$$

$$\Rightarrow \boxed{n \geq 18.93156857} \Rightarrow \boxed{n \geq 19}$$

~~we need~~

\therefore we need 19 iteration or larger to have
an accuracy with 10^{-6}

$$f(2.5) = (2.5)^2 - 2(2.5) - 1$$

$$6.25 - 5 - 1 = 0.25$$

(15 points)

Question 4:

Let $f(x) = x^2 - 2x - 1$

a) Use Newton-Raphson method with $P_0 = 2.5$ to find P_3 .

b) Use the secant method with $P_0 = 2.6$, $P_1 = 2.5$ to find P_3 .

② $P_n = P_{n-1} - \frac{f(P_{n-1})}{f'(P_{n-1})}$

$$f(x) = x^2 - 2x - 1$$

$$f'(x) = 2x - 2$$

$$P_1 = P_0 - \frac{f(P_0)}{f'(P_0)} \Rightarrow P_1 = 2.5 - \frac{f(2.5)}{f'(2.5)} = 2.5 - \frac{0.25}{3}$$

$$= 2.416666667$$

$$\approx 2.417$$

$$P_2 = P_1 - \frac{f(P_1)}{f'(P_1)} = 2.417 - \frac{f(2.417)}{f'(2.417)} = 2.417 - \frac{0.007889}{2.834}$$

$$= 2.414216302$$

$$\approx 2.414$$

$$P_3 = P_2 - \frac{f(P_2)}{f'(P_2)} = 2.414 - \frac{f(2.414)}{f'(2.414)} = 2.414 - \frac{0.000604}{2.828}$$

$$= 2.414213579$$

$$\approx 2.414$$

Good Luck

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$$x^2 - 2x - 1$$

$$\textcircled{b} \quad P_n = P_{n-1} - \frac{f(P_{n-1}) [P_{n-2} - P_{n-1}]}{[f(P_{n-2}) - f(P_{n-1})]}$$

$$P_2 = P_1 - \frac{f(P_1) [P_0 - P_1]}{[f(P_0) - f(P_1)]}$$

$$P_2 = 2.5 - \frac{f(2.5) [2.6 - 2.5]}{f(2.6) - f(2.5)}$$

$$= 2.5 - \frac{0.25 [0.1]}{0.56 - 0.25} = 2.5 - \frac{0.025}{0.31}$$

$$\text{~~2.5~~} = 2.419354839 \approx \boxed{2.419}$$

$$P_3 = P_2 - \frac{f(P_2) [P_1 - P_2]}{f(P_1) - f(P_2)}$$

$$= 2.419 - \frac{f(2.419) [2.5 - 2.419]}{f(2.5) - f(2.419)}$$

$$= 2.419 - \frac{0.013561 [0.081]}{0.25 - 0.013561}$$

$$= 2.419 - \frac{0.001098441}{0.236439}$$

$$= 2.414354231$$

$$\approx \boxed{2.414}$$

تم بحمد الله

Finished

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